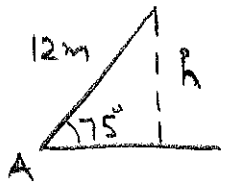
Ambiguous Case of Sine Law

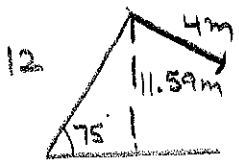
1. a)



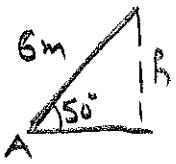
$h = (12)(\sin 75^\circ) = (12)(0.96) = 11.59\text{ m}$

we are given that $a = 4$

As $a < h \Rightarrow$ No $\triangle$ possible



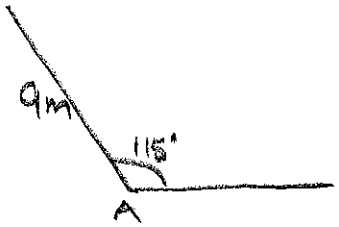
b)



We are given that $a = 10\text{m}$, $b = 6\text{m}$

As $a > b \Rightarrow$ One $\triangle$ possible

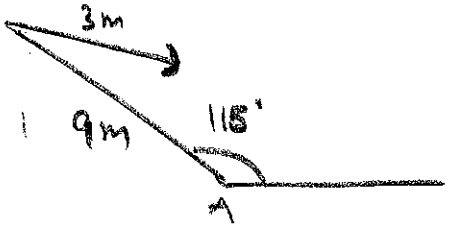
c)

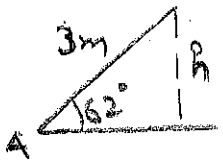


We are given that $a = 3\text{m}$, $b = 9\text{m}$ and $\angle A = 115^\circ$ (which is obtuse)

As $\angle A$ is obtuse, and $a < b$

$\Rightarrow$ No $\triangle$ possible

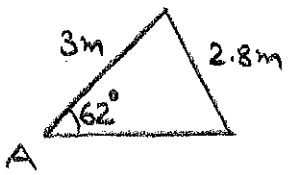


1d) 

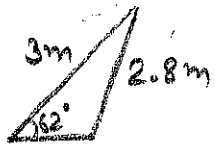
$h = (3)(\sin 62^\circ) = (3)(0.88) = 2.6\ m$

We are given that $a = 2.8\ m$, $b = 3$

As $a > h$, $a < b$ $\Rightarrow$ Two Δs possible



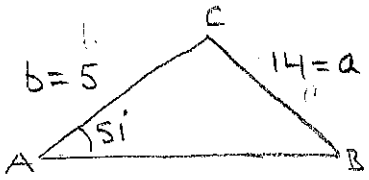
↑
Situation I



↑
Situation II

Q.2 a Angle = 51°, Side One = 5cm, Side Two = 14cm

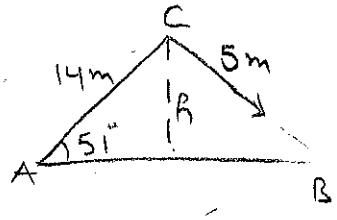
Case I



As $a > b$

One Δ possible

Case II



$h = (14)\sin 51^\circ$

$h = 10.88\ m$

As $a < h$

No Δ possible

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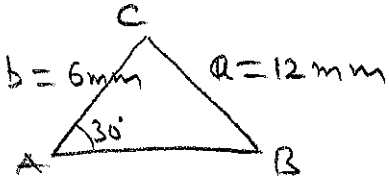
b) Angle = 30°, side One = 6mm, Side Two = 12mm

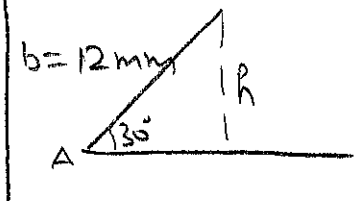
Case I



As $a > b$

One Δ possible

Case II

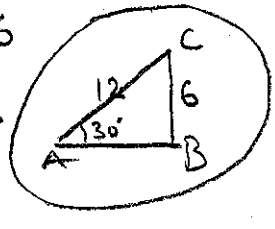


$h = (12)(\sin 30^\circ)$

$h = 6$

As $a = 6$, and $h = 6$

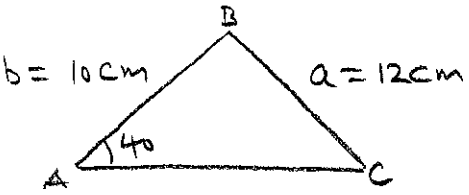
One Δ possible →



2c

Angle = 40°, Side One = 12cm, Side Two = 10cm

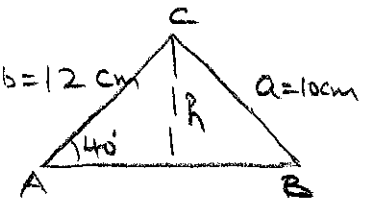
Case I



As $a > b$

One Δ possible

Case II

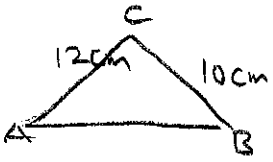


$h = (12)(\sin 40°)$

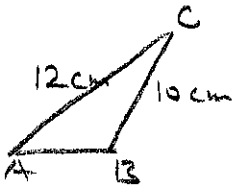
$h = 7.71\,cm$

As $a > h$, $a < b$

Two Δs possible

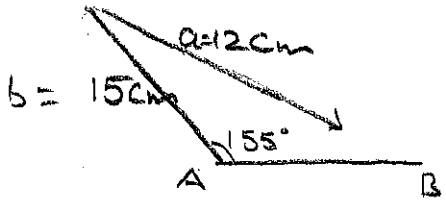


&



2d Angle = 155°, side one = 15cm, side Two = 12cm

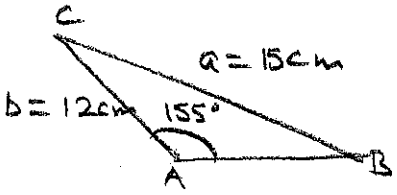
Case I



As $a < b$ (Use obtuse Δ rules)

No Δ possible

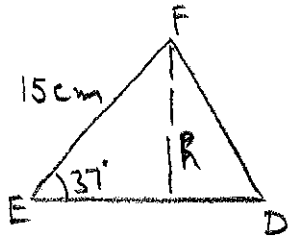
Case II



As $a > b$ (Use obtuse Δ rules)

One Δ possible

5. a)



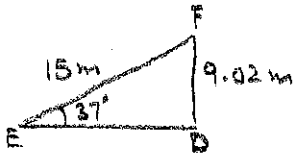
$h = (15)(\sin 37°) = 9.02\text{ m}$

b) No $\triangle$ possible $\Rightarrow$ $FD < h$

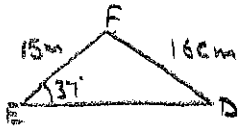
$\Rightarrow$ $FD = 7\text{m}$

One $\triangle$ possible $\Rightarrow$ $FD = h$ OR $FD > EF$

$\Rightarrow$ $FD = 9.02\text{m}$

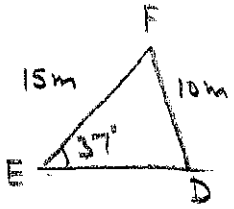


$\Rightarrow$ $FD = 16\text{cm}$

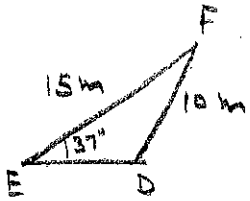


Two $\triangle$s possible $\Rightarrow$ $FD > h$ and $FD < EF$

$\Rightarrow$ $FD = 10\text{m}$

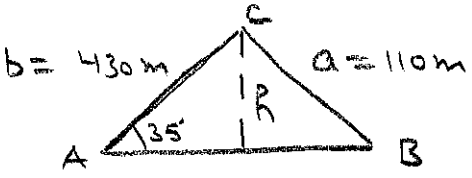


and



6. 183

Situtation I

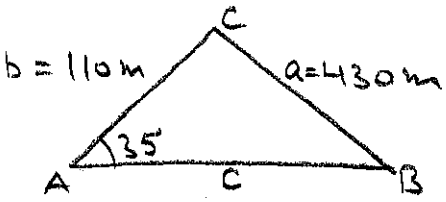


$h = (430m)(\sin 35^\circ) = 246.63m$

As $a < h$

No Δ possible

Situtation II



As $a > b$

One Δ is possible

Step. 1. $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{110} = \frac{\sin 35^\circ}{430}$

$\sin B = \frac{(110)(\sin 35^\circ)}{430}$

$\sin B = 0.1467$

$\angle B = \sin^{-1}(0.1467)$

$\angle B = 8.4^\circ$

Step. 2. $\angle C = 180^\circ - (35^\circ + 8.4^\circ) = 136.6^\circ$

Step. 3. $c^2 = a^2 + b^2 - 2ab\cos C$

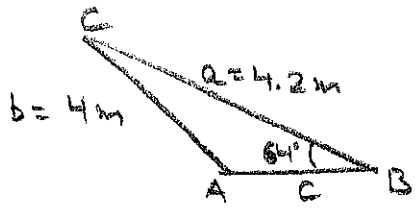
$c^2 = (430)^2 + (110)^2 - 2(430)(110)\cos 136.6^\circ$

$c^2 = 184900 + 12100 + 68733.9639$

$c^2 = 265733.9639$

$C = 515\ m$

89
184



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{4.2} = \frac{\sin 64^\circ}{4}$$

$$\sin A = \frac{(4.2)(\sin 64^\circ)}{4}$$

$$\sin A = 0.9437$$

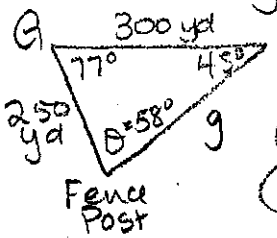
$$\sin^{-1}(\sin A) = \sin^{-1}(0.9437)$$

$$\angle A = 70.68^\circ$$

$$\angle A = 180^\circ - 70.68^\circ = 109.3^\circ$$

b) There is only one possible Δ, because if we interchange 'a' and 'b', there will be no Δ possible.

10. acute triangle



east than south west means a 45° angle

① $\frac{\sin 45^\circ}{250} = \frac{\sin \theta}{300}$

$\frac{300 \sin 45^\circ}{250} = \sin \theta$

$\frac{300(0.7071)}{250} = \sin \theta$

$0.8485 = \sin \theta$

$\theta = 58^\circ$

② $\angle G = 180^\circ - 45^\circ - 58^\circ$

$\angle G = 77^\circ$

③ $g^2 = 250^2 + 300^2 - 2(250)(300)\cos 77^\circ$

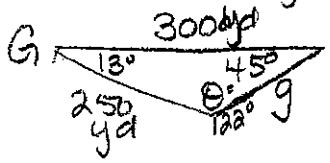
$g^2 = 62500 + 90000 - 33742.6582$

$g^2 = 118757.3418$

$g = 345 \text{ m}$

P. 183 cont.

10. cont. obtuse triangle

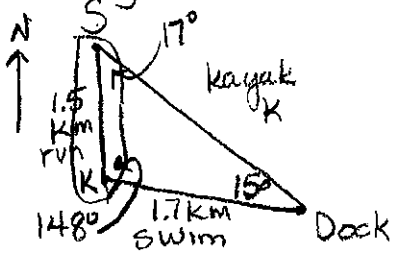


① $\theta = 180 - 58° = 122°$

② $\angle G = 180° - 122° - 45°$
$\angle G = 13°$

③ $g^2 = 250^2 + 300^2 - 2(250)(300)\cos 13°$
$g^2 = 62500 + 90000 - 146155.5097$
$g^2 = 6344.4903$
$g = 80$ m

11.



① $\frac{\sin S}{1.7} = \frac{\sin 15°}{1.5}$

$\sin S = \frac{1.7 \sin 15°}{1.5}$

$\sin S = \frac{1.7(0.2588)}{1.5}$

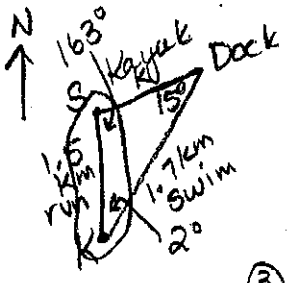
$\sin S = 0.2933$

$\angle S = 17°$

② $\angle K = 180° - 15° - 17°$
$\angle K = 148°$

③ $k^2 = 1.5^2 + 1.7^2 - 2(1.5)(1.7)\cos 148°$
$k^2 = 2.25 + 2.89 + 4.3250$
$k^2 = 9.465$
$k = 3.1$ km

OR



① $\angle S = 180° - 17° = 163°$

② $\angle K = 180° - 163° - 15° = 2°$

③ $k^2 = 1.5^2 + 1.7^2 - 2(1.5)(1.7)\cos 2°$
$k^2 = 2.25 + 2.89 - 5.09689$
$k^2 = 0.04311$
$k = 0.2$ km