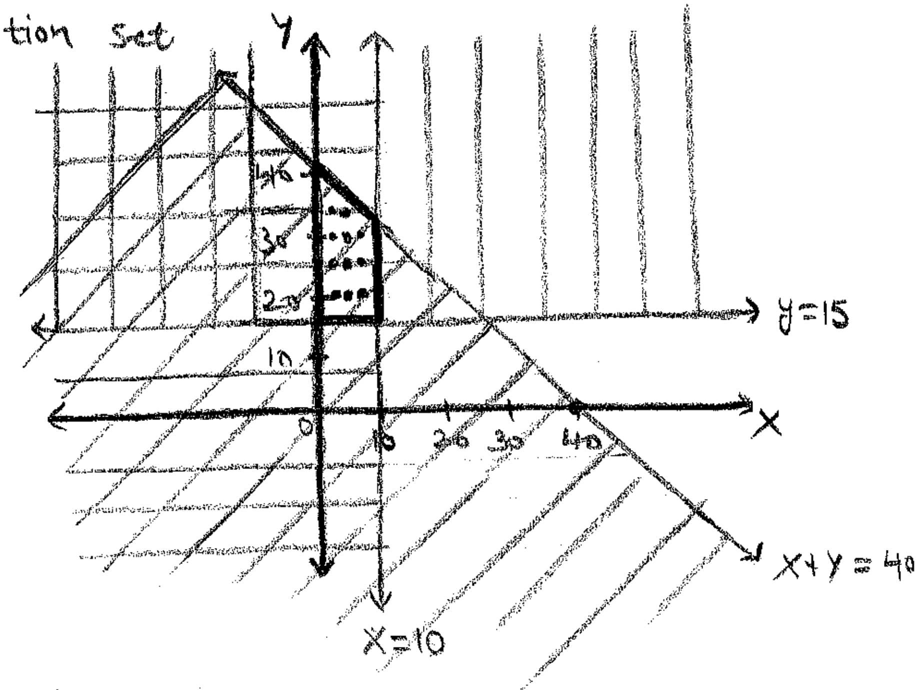
Q.1. <u>Step. 1.</u> Variables → X = number of Canadian songs
Y = number of American songs

<u>Step. 2.</u> Restrictions → X & Y both are whole numbers → $X \,\&\, Y \in W$

<u>Step. 3.</u> Inequalities → $X \leq 10$ (10 or fewer Canadian songs)
$Y \geq 15$ (15 or more American songs)
$X + Y \leq 40$ (40 or fewer songs, in total)

<u>Step. 4.</u> Graph the solution set



<u>Step. 5.</u> Let C represents the cost.
The objective function is

$$C = (\$1.50)X + (\$.75)Y$$

<u>Step. 6.</u> The solution region has four vertices (0,15), (0,40), (10,15), (10,30)

If $(x,y) \to (0,15)$	If $(x,y) \to (0,40)$	If $(x,y) \to (10,15)$	If $(x,y) \to (10,30)$
$C = (\$1.50)(0) + (\$.75)(15)$			
$C = 0 + \$11.25$		$C = 15 + 11.25$	$C = 15 + 22.5$
$C = \$11.25$	$C = \$30$	$C = \$26.25$	$C = \$37.50$

<u>Step 7.</u> Josh can minimize the cost to \$11.25 by downloading zero Canadian songs and 15 American songs and maximize cost to \$ 37.50 by downloading 10 Canadian and 30 American songs.

Q.2. Step. 1. Variabes → X = Number of hours at first job.
Y = Number of hours at second job.

Step. 2. Restrictions → X & Y both are whole numbers → $X \& Y \in W$

Step. 3. Inequalities → $X+Y \leq 32$ (No more than 32h, in total)

$X \geq 12$ (No less than 12h at one job)

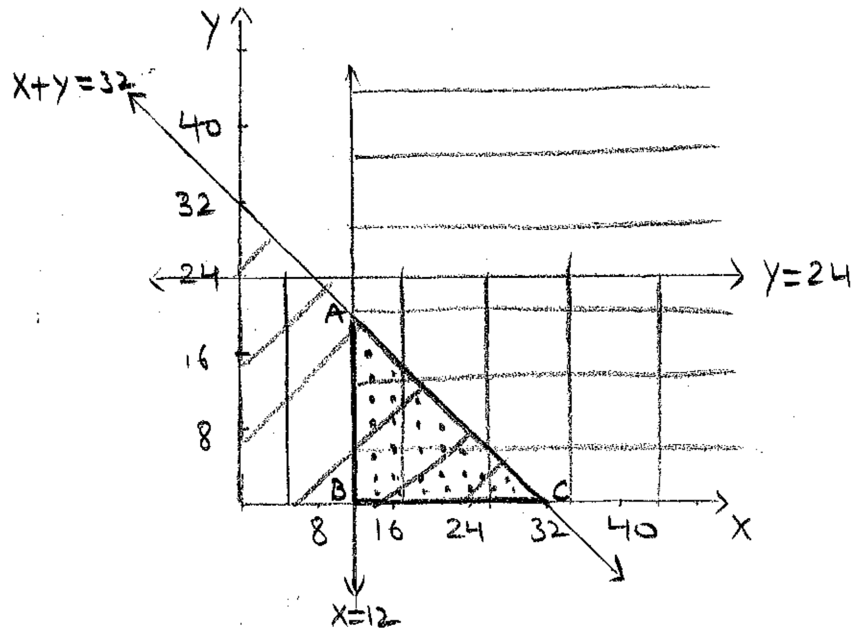
$Y \leq 24$ (No more than 24h at other job)

Step. 4. Graph

$X \geq 12$
$X = 12 \rightarrow$ x-int

$Y \leq 24$
$Y = 24 \rightarrow$ y-int

$X+Y \leq 32$
$X+Y = 32$
x-int → 32
y-int → 32



Test

$X \geq 12$
$0 \geq 12$
False
Sol. on the other side as (0,0)

$Y \leq 24$
$0 \leq 24$
True
Sol. on the same side as (0,0)

$X+Y \leq 32$
$0+0 \leq 32$
$0 \leq 32$
True

Step. 5. Let E represents the earnings.
The objective function is
$E = X(11.50) + Y(12)$

Step. 6. The solution region has Three vertices A(12,20), B(12,0), C(32,0)

If $(x,y) \rightarrow (12,20)$
$E = X(11.50) + Y(12)$
$E = (12)(11.50) + (20)(12)$
$E = 138 + 240$
$E = \$378$

If $(x,y) \rightarrow (12,0)$
$E = X(11.50) + Y(12)$
$E = (12)(11.50) + (0)(12)$
$E = 138 + 0$
$E = \$138$

If $(x,y) \rightarrow (32,0)$
$E = X(11.50) + Y(12)$
$E = (32)(11.50) + (0)(12)$
$E = 368 + 0$
$E = \$368$

Step. 7. Jenny can maximize her earnings to \$ 378 by working 12h at first job and 20h at second job. She can minimize her earning to \$138 by working 12h at first job and 0h at second job.

Q.3 Step. 1. Variables → X = Number of wide boards
Y = Number of narrow boards

Step. 2. Restrictions → X & Y both are whole numbers → X & Y $\in W$

Step. 3. Inequalities →

$X \geq 100$ (No fewer than 100 wide boards)

$Y \leq 80$ (No more than 80 narrow boards)

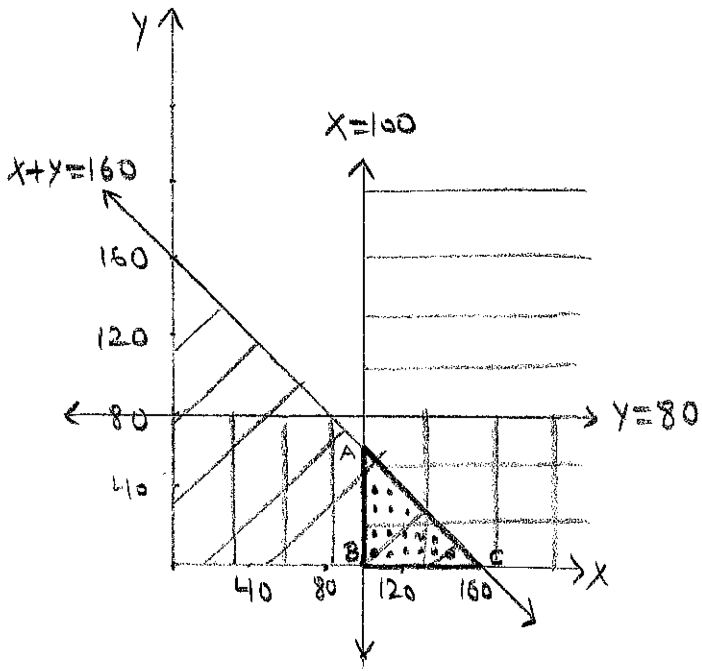
$X + Y \leq 160$ (No more than 150 boards, in total)

Step 4 Graph

$X \geq 100$
$x = 100$ → X-intercept

$Y \leq 80$
$y = 80$ → Y-intercept

$X + Y \leq 160$
$X + Y = 160$
$X = 160$, $Y = 160$
↑ X-int ↑ Y-int



Test

$X \geq 100$
$0 \geq 100$
False
Sol on the other side as (0, 0)

$Y \leq 80$
$0 \leq 80$
True
Sol on the same side as (0, 0)

$X + Y \leq 160$
$0 + 0 \leq 160$
$0 \leq 160$
True
Sol on the same side as (0, 0)

Step. 5. Let C represents the cost of the fence.

$C = X(4.36) + Y(3.56)$ → objective function

Step. 6. The solution region has three vertices A(100, 60), B(100, 0) and C(160, 0)

If $(x, y) \to (100, 60)$
$C = X(4.36) + Y(3.56)$
$C = (100)(4.36) + (60)(3.56)$
$C = 436 + 213.6$
$C = \$649.6$

If $(x, y) \to (100, 0)$
$C = X(4.36) + Y(3.56)$
$C = (100)(4.36) + (0)(3.56)$
$C = 436 + 0$
$C = \$436$

If $(x, y) \to (160, 0)$
$C = X(4.36) + Y(3.56)$
$C = (160)(4.36) + (0)(3.56)$
$C = 697.6 + 0$
$C = \$697.6$

Step. 7. Maximum cost is \$697.6 with '160' wide boards and '0' narrow ones.

Minimum cost is \$436 with '100' wide boards and '0' narrow ones.

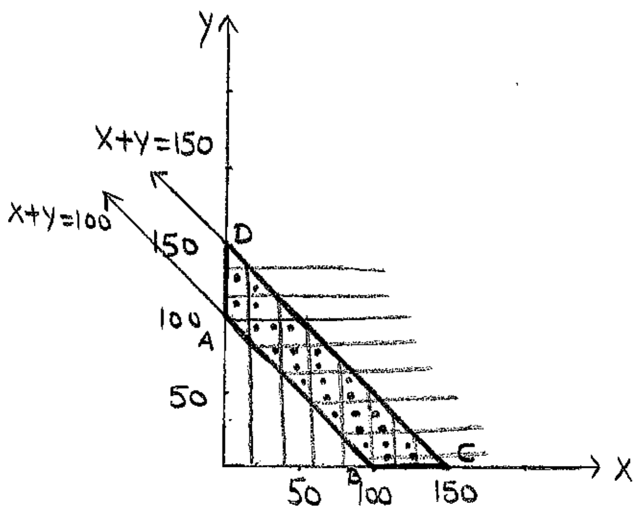
Q.4. Step 1. Variables → X = Number of track events
Y = Number of field events

Step 2. Restrictions → Both X & Y are whole numbers → $X \in W$, $Y \in W$

Step 3. Inequalities → $X+Y \leqslant 150$ (No more than 150 events, all together)
$X+Y \geqslant 100$ (No fewer than 100 events, all together)

Step 4 Graph

$X+Y \leqslant 150$
X-intercept → 150
Y-intercept → 150
$X+Y \geqslant 100$
X-intercept → 100
Y-intercept → 100



Test
$X+Y \leqslant 150$
$0+0 \leq 150$
$0 \leq 150$
True
⇒ Sol on same side as (0,0)
$X+Y \geqslant 100$
$0+0 \geqslant 100$
$0 \geqslant 100$
False
⇒ sol on the other side as (0,0)

Step 5 Let T represents the time in hours
$\Rightarrow T = X(.25) + Y(.50)$

Step 6. The solution region has four vertices A(0,100), B(100,0), C(150,0), D(0,150)

If $(x,y) \to (0,100)$	If $(x,y) \to (100,0)$	If $(x,y) \to (150,0)$	If $(x,y) \to (0,150)$
$T = X(.25) + Y(.50)$	$T = X(.25) + Y(.50)$	$T = X(.25) + Y(.50)$	$T = X(.25) + Y(.50)$
$T = (0)(.25) + (100)(.50)$	$T = (100)(.25) + (0)(.50)$	$T = (150)(.25) + (0)(.50)$	$T = (0)(.25) + (150)(.50)$
$T = 0 + 50$	$T = 25 + 0$	$T = 37.5 + 0$	$T = 0 + 75$
$T = 50h$	$T = 25h$	$T = 37.5h$	$T = 75h$

Step. 7. Maximum time of 75h with the combination of '0' track events and '150' field events.
Minimum time of 25h with the combination of '100' track events and '0' field events.