6.4
Optimisation Problems

Ex.1. A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

Sol.

Step.1. Define the variables.

Let s represent sport-utility vehicles.
Let r represent racing cars.

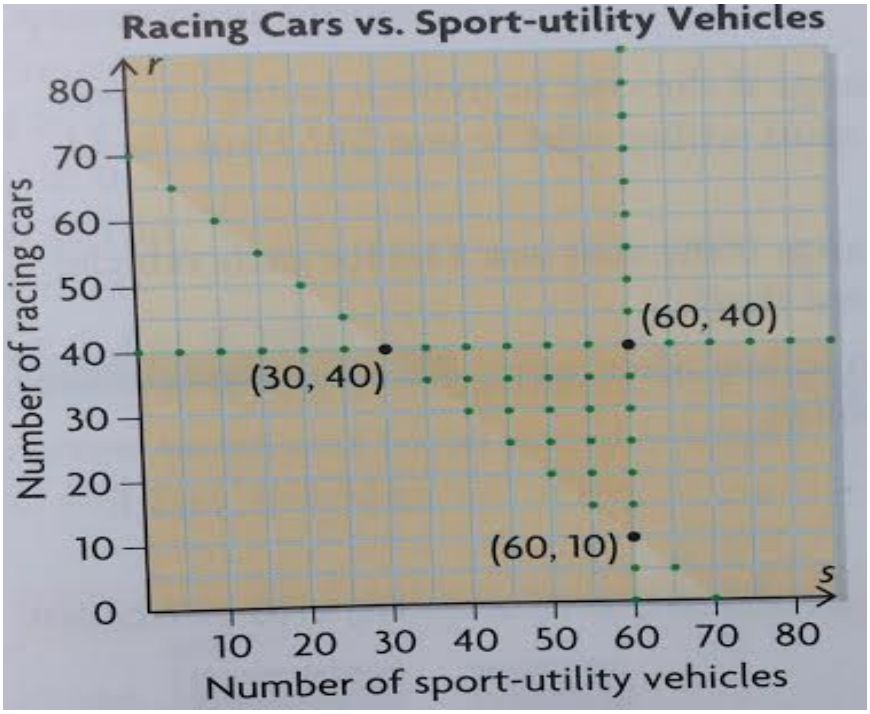
Step.2.Describe the restrictions on the variables in this situation. (Domain & Range)

$r \in W$, $s \in W$

Step.3.Write a system of inequalities that models this situation.

$r \leq 40$
$s \leq 60$
$r + s \geq 70$

Step.4. Graph the system to determine the coordinates of the vertices of its feasible region.



Step.5. Write an objective function that shows how the variables are related to the quantity to be optimised.

Let C represents the total production cost.
The objective function to optimize, at $12 per sport-utility and $8 per car:
C = 12s + 8r

Step.6. Evaluate the objective function by substituting the values of the coordinates of each vertex.

Objective function
C = 12s + 8r

If (s, r) is (60, 10), $C = 12(60) + 8(10)$ $C = 720 + 80$ $C = \$800$	If (s, r) is (60, 40), $C = 12(60) + 8(40)$ $C = 720 + 320$ $C = \$1040$	If (s, r) is (30, 40), $C = 12(30) + 8(40)$ $C = 360 + 320$ $C = \$680$

Step.7.Compare the results and choose the desired solution.

The company can minimize the production cost to $680 by making 30 sport- utility vehicles and 40 racing cars and maximize costs to $1040 by making 60 sport- utility vehicles and 40 racing cars.

Practice

Q.1. Josh wants to download different kinds of songs to his ipod. He wants to download 10 or fewer Canadian songs, and 15 or more American songs. He wants to download 40 or fewer songs, in total.
A Canadian song costs $1.50 and an American song costs $.75. What combinations of songs will allow him to spend minimum and maximum money?

Sol.

Step.1. Define the variables.
Step.2. Describe the restrictions on the variables in this situation. (Domain & Range)
Step.3. Write a system of inequalities that models this situation.
Step.4. Graph the system to determine the solution set.
Step.5. Write an objective function that shows how the variables are related to the quantity to be optimised.
Step.6. Evaluate the objective function by substituting the values of the coordinates of each vertex.
Step.7. Compare the results and choose the desired solution.

Q.2. Jenny has two summer jobs .

- She works no more than a total of 32 h a week.
- At one job, she works no less than 12 h and earns $11.50/h.
- At other job, she works no more than 24 h and earns $12/h.

What combinations of number of hours will allow her to minimise and maximise her earnings? What can she expect to earn?

Sol.

Step.1. Define the variables.
Step.2. Describe the restrictions on the variables in this situation. (Domain & Range)
Step.3. Write a system of inequalities that models this situation.
Step.4. Graph the system to determine the solution set.
Step.5. Write an objective function that shows how the variables are related to the quantity to be optimised.
Step.6. Evaluate the objective function by substituting the values of the coordinates of each vertex.
Step.7. Compare the results and choose the desired solution.

Q.3. Jack's Company is competing for a contract to build a fence. The company will be using two kinds of boards (wide boards and narrow boards) to build fence.

- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The fence will include no more than 160 boards, in total.
- Each wide board costs $4.36 whereas each narrow board costs $ 3.56 .

Determine the maximum and minimum costs to build the fence.

Sol.

Step.1. Define the variables.
Step.2. Describe the restrictions on the variables in this situation. (Domain & Range)
Step.3. Write a system of inequalities that models this situation.
Step.4. Graph the system to determine the solution set.
Step.5. Write an objective function that shows how the variables are related to the quantity to be optimised.
Step.6. Evaluate the objective function by substituting the values of the coordinates of each vertex.
Step.7. Compare the results and choose the desired solution.

Q.4. A school is organising a track and field meet.

- There will be no more than 150 events and no fewer than 100 events.
- The organisers will allow 15 min (.25 h) for each track event and 30 min (.50 h) for each field event.

What combinations of track and field events will consume minimum and maximum time?

Sol.

Step.1. Define the variables.
Step.2. Describe the restrictions on the variables in this situation. (Domain & Range)
Step.3. Write a system of inequalities that models this situation.
Step.4. Graph the system to determine the solution set.
Step.5. Write an objective function that shows how the variables are related to the quantity to be optimised.
Step.6. Evaluate the objective function by substituting the values of the coordinates of each vertex.
Step.7. Compare the results and choose the desired solution.